MATDIP301

USN

Third Semester B.E Degree Examination, July/August 2003

Common to All Branches

Advanced Mathematics - I

Time: 3 hrs.] [Max.Marks : 100

Note: *(i) Answer any FIVE full questions.*
(ii) All questions carry equal marks

1. (a) Find the n^{th} order derivaties of (i) $e^{ax}cos\ (bx+c)$ (ii) $log\ (ax+b)$ (5+5=10 Marks)

   (b) Compute the n^{th} derivative of cosx cos2x cos3x. (5 Marks)

   (c) If $y = sin^{-1}x$ prove that $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - n^2/n = 0$ (5 Marks)

2. (a) With usual notation prove that

   i) $tan\ \Phi = \frac{rd\theta}{dr}$ ii) $\frac{1}{p^2} = \frac{1}{r^2} + \frac{1}{r^4}\left(\frac{dr}{d\theta}\right)^2$. (5+5 Marks)

   (b) Prove that the following pairs of curve intersect orthogonally.

   $r^n = a^n cos n\theta,\quad r^n = b^n sin n\theta.$ (5 Marks)

   (c) Find the pedal equation of the polar curve

   $\frac{2a}{r} = 1 - cos\theta.$ (5 Marks)

3. (a) If $U = cot^{-1}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right)$ prove that

   $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = -\frac{1}{4}$ sin 2u. (5 Marks)

   (b) If $U = log(x^3+y^3+z^3-3xyz)$ prove that

   $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = \frac{3}{x+y+z}$. (5 Marks)

   (c) Prove that $JJ' = 1$. (5 Marks)

   (d) If Z = f(u.v) where

   $u = x^2 - y^2$ and $v = 2xy$

   Prove that : $x\frac{\partial z}{\partial x} - y\frac{\partial z}{\partial y} = 2(x^2+y^2)\frac{\partial z}{\partial u}$ (5 Marks)

Contd.... 2

5. (a) Prove that $\Gamma\frac{1}{2} = \sqrt{\pi}$ **(6 Marks)**

(b) Show that $\beta(m,n) = 2\int_0^{\pi/2} \cos^{2m-1}\theta \sin^{2n-1}\theta \, d\theta$. **(7 Marks)**

(c) Show that : $\int_a^b (x-a)^{m-1}(b-x)^{n-1}\,dx = (b-a)^{m+n-1}\beta(m,n)$ **(7 Marks)**

6. (a) Solve : $\frac{dy}{dx} = \frac{x(2\log x+1)}{(\sin y + y\cos y)}$ **(5 Marks)**

(b) Solve : $\frac{dy}{dx} = \frac{y+x-2}{y-x-4}$ **(5 Marks)**

(c) Solve : $\frac{dy}{dx} + x\sin 2y = x^3\cos^2 y$ **(5 Marks)**

(d) Solve : $(3x^2 + 6xy^2)dx + (6x^2y + 4y^3)dy = 0$ **(5 Marks)**

7. (a) Solve : $\frac{d^4x}{dt^4} + 4x = 0$ **(6 Marks)**

(b) Solve : $y'' - 3y' + 2y = e^{3x}$ **(7 Marks)**

(c) Solve : $\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = 3x + 4$ **(7 Marks)**

8. (a) Reduce $1 - \cos\alpha + i\sin\alpha$ to the modulus amplitude form. **(6 Marks)**

(b) Express $\frac{(1+i)(2+i)}{(3+i)}$ in the form a + ib. **(7 Marks)**

(c) If $\sin\alpha + \sin\beta + \sin\gamma = \cos\alpha + \cos\beta + \cos\gamma = 0$
prove that :

i) $\sin 3\alpha + \sin 3\beta + \sin 3\gamma = 3\sin(\alpha+\beta+\gamma)$

ii) $\cos 3\alpha + \cos 3\beta + \cos 3\gamma = 3\cos(\alpha+\beta+\gamma)$ **(7 Marks)**

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MATDIP301

USN

Third Semester B.E Degree Examination, January/February 2004

Common to All Branches

(Old Scheme)

Advanced Mathematics - I

Time: 3 hrs.] [Max.Marks : 100

Note: *(i) Answer any FIVE full questions.*
(ii) All questions carry equal marks

1. (a) Find nth derivatives of
   i) $log(ax+b)$ ii) $sin3xcos2x$ (5+5 Marks)

   (b) If $y = x^3e^{2x}$, find y_n (5 Marks)

   (c) If $y = cos(mlogx)$ then prove that
   $x^2y_{n+2} + (2n+1)xy_{n+1} + (m^2+n^2)y_n = 0$ (5 Marks)

2. (a) With the usual notations prove that;
   i) $tan\phi = r\frac{d\theta}{dr}$
   and hence prove that
   ii) $\frac{1}{p^2} = \frac{1}{r^2} + \frac{1}{r^4}(\frac{dr}{d\theta})^2$ (5+5 Marks)

   (b) Find the angle of intersection between the curves
   $r = a(1-cos\theta)$; $r = 2acos\theta$ (5 Marks)

   (c) Find Pedal equation of $\frac{2a}{r} = 1 - cos\theta$ (5 Marks)

3. (a) If $f = f(x,y)$ is homogeneous function of degree 'n' then prove that
   i) $x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y} = nf$ and
   ii) $x^2\frac{\partial^2 f}{\partial x^2} + y^2\frac{\partial^2 f}{\partial y^2} + 2xy\frac{\partial^2 f}{\partial x\partial y} = n(n-1)f$ (5+5 Marks)

   (b) If $x = rsin\theta cos\phi$; $y = rsin\theta sin\phi$; $z = rcos\theta$ find $\frac{\partial(x,y,z)}{\partial(r,\theta,\phi)}$ (5 Marks)

   (c) If $U = f(r)$ where $r = \sqrt{x^2+y^2+z^2}$, then prove that
   $U_{xx} + U_{yy} + U_{zz} = f''(r) + \frac{2}{r}f'(r)$ (5 Marks)

4. (a) Obtain reduction formula for $\int_0^{\pi/2} sin^n x dx$, where 'n' is positive integer.
   Also evaluate: $\int_0^{\frac{\pi}{2}} sinxdx$ (5+5 Marks)

   (b) Evaluate: $\int_0^1 \int_0^x e^{y/x}dydx$ (5 Marks)

   (c) Evaluate: $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} xyzdzdydx$ (5 Marks)

Contd.... 2

5. (a) With usual notation prove that (10 Marks)

$$\Gamma(m)\Gamma(m+\tfrac{1}{2}) = (\sqrt{\pi}\,\Gamma(2m)) \,/\, 2^{2m-1}$$

(b) Evaluate $\int_0^{\pi/2} \sqrt{cot\theta}\, d\theta$ (5 Marks)

(c) Using Beta and Gamma functions, evaluate

$$\int_0^{\frac{\pi}{2}} sin^4\theta \; cos^5\theta \; d\theta$$ (5 Marks)

6. (a) Solve $\frac{dy}{dx} = e^{x-y} + x^2 e^{-y}$ (5 Marks)

(b) Solve $x^2 y dx = (x^3 + y^3) dy$ (5 Marks)

(c) Solve $x\frac{dy}{dx} + y = x^3 y^6$ (5 Marks)

(d) Solve $(1 + e^{x/y})dx + e^{x/y}(1 - x/y)\, dy = 0$ (5 Marks)

7. Solve the following differential equations:

(a) $y'' + 5y' + 6y = e^{2x}$ (5 Marks)

(b) $y'' - 2y' + y = x^2 e^x$ (5 Marks)

(c) $y'' + y' - 2y = x + sinx$ (5 Marks)

(d) $y'' - 2y' + 2y = e^x cos2x$ (5 Marks)

8. (a) Express $1 + cos\alpha + isin\alpha$ in modulus and amplitude form. (5 Marks)

(b) Prove that

$$(1 + cos\theta + isin\theta)^n + (1 + cos\theta - isin\theta)^n = 2^{n+1} cos^n \tfrac{\theta}{2} cos(\tfrac{n\theta}{2})$$ (5 Marks)

(c) Find the different values $(1+i)^{\frac{1}{3}}$ (5 Marks)

(d) If $sin\alpha + sin\beta + sin\gamma = cos\alpha + cos\beta + cos\gamma = 0$, prove that $sin(3\alpha) + sin(3\beta) + sin(3\gamma) = 3sin(\alpha + \beta + \gamma)$ (5 Marks)

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MATDIP301

USN

Third Semester B.E Degree Examination, July/August 2004

Common to All Branches

Advanced Mathematics - I

Time: 3 hrs.] [Max.Marks : 100

Note: *(i) Answer any FIVE full questions.*
(ii) All questions carry equal marks.

1. (a) Find the n^{th} derivative of $sin(ax+b)$ (6 Marks)

   (b) Obtain the n^{th} derivative of $y = \frac{x}{(x-1)(2x+3)}$ (7 Marks)

   (c) If $y = e^{asin^{-1}x}$ prove that
   $$(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - (n^2+a^2)y_n = 0$$ (7 Marks)

2. (a) With usual notation prove that $tan\phi = r\frac{d\theta}{dr}$ (6 Marks)

   (b) Find the pedal equation of the curve $2a = r(1+cos\theta)$ (7 Marks)

   (c) Expand $log(1+e^x)$ in powers of x by Maclaurin's theorem upto x^3 (7 Marks)

3. (a) If $u = e^{x^3+y^3}$ prove that
   $$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 3ulogu$$ (6 Marks)

   (b) Verify Eulers theorem for the function
   $$u = (x^{\frac{1}{2}} + y^{\frac{1}{2}}).(x^n + y^n)$$ (7 Marks)

   (c) If $x = rcos\theta,\ y = rsin\theta, z = z$ find
   $\frac{\partial(x,y,z)}{\partial(r,\theta,z)}$ (7 Marks)

4. (a) Find the Reduction formula for $I_n = \int cosec^n x dx$ (6 Marks)

   (b) Find the value of $\int_0^1 x^2(1-x^2)^{3/2}dx$ using Reduction formula. (7 Marks)

   (c) Prove that $\int_{-1}^{1} dz \int_0^z dx \int_{x-z}^{x+z} (x+y+z)dy = 0$ (7 Marks)

5. (a) Prove that $\beta(m,n) = 2\int_0^{\frac{\pi}{2}} sin^{2m-1}\theta\ cos^{2n-1}\theta d\theta$ (6 Marks)

   (b) Show that $\int_0^{\pi/2} \sqrt{tan\theta}d\theta \times \int_0^{\pi/2} \sqrt{cot\theta}d\theta = \frac{\pi^2}{2}$ (7 Marks)

   (c) Show that $\int_0^{\infty} \frac{x^8(1-x^6)}{(1+x)^{24}}dx = 0$ (7 Marks)

Contd.... 2

6. (a) Solve $(x+1)\frac{dy}{dx}+1=2e^{-y}$ (5 Marks)

(b) Solve $(2x-6y+7)dx=(x-3y+4)dy$ (5 Marks)

(c) Solve $\frac{dy}{dx}+ycotx=4xcosecx$ (5 Marks)

(d) Solve $ysin2xdx-(1-y^2+cos^2x)dy=0$ (5 Marks)

7. (a) Solve $\frac{d^3y}{dx^3}-6\frac{d^2y}{dx^2}+11\frac{dy}{dx}-6y=0$ (6 Marks)

(b) Find the solution of

$$[D^2-13D+12]y=e^{2x}+5e^x$$ (7 Marks)

(c) Solve $\frac{d^2y}{dx^2}-3\frac{dy}{dx}+2y=cos2x$ (7 Marks)

8. (a) Show that if n is a positive integer, then

$$\left(\frac{1+sin\theta+icos\theta}{1+sin\theta-icos\theta}\right)^n=cosn(\frac{\pi}{2}-\theta)+isinn(\frac{\pi}{2}-\theta)$$ (6 Marks)

(b) Express $sin^4\theta cos^3\theta$ in terms of cosines of multiples of θ (7 Marks)

(c) Express $\sqrt{3}+i$ in the polar form and hence find their modulus and amplitude. (7 Marks)

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MATDIP301

USN | | | | | | | | | | |

rd Semester B.E Degree Examination, January/February 2005

Common to All Branches

(Old Scheme)

Advanced Mathematics - I

3 hrs.] [Max.Marks : 100

Note: *(i) Answer any FIVE full questions.*
(ii) All questions carry equal marks

1. (a) Find the n^{th} derivatives of

   i) $\frac{x^2}{(2x+1)(2x+3)}$ ii) $e^{2x}\cos^2 x \sin x$

   (b) State Leibnitz's theorem for the n^{th} derivative of a product of two functions. Find the n^{th} derivative of $x^2 \log 4x$.

   (c) If $y = e^{m\sin^{-1}x}$

   Show :

   i) $(1-x^2)y_2 - xy_1 = m^2y$

   ii) $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - (n^2+m^2)y_n = 0$ (7+7+6 Marks)

2. (a) With the usual notation, prove that $\tan\phi = r\frac{d\theta}{dr}$.

   (b) Using Maclaurin's series, expand $\log(1+\sin x)$ upto the term containing x^4.

   (c) Find the pedal equation of the curve $r^m = a^m \cos m\theta$. (7+7+6 Marks)

3. (a) If $u = x^2+y^2+z^2$ and $x = e^{2t}, y = e^{2t}\cos 3t$ $z = e^{2t}\sin 3t$. Find $\frac{du}{dt}$ as a total derivative and verify the result by direct substitution.

   (b) If $x = r\cos\theta$, $y = r\sin\theta$, then find $J\left(\frac{x,y}{r,\theta}\right)$ and $J'\left(\frac{r,\theta}{x,y}\right)$. Verify that $J.J' = 1$.

   (c) If u is a homogeneous function of degree n in x and y, then prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = nu$. (7+7+6 Marks)

4. (a) Obtain the reduction formula for $I_n = \int_0^{\frac{\pi}{2}} \sin^n x\, dx$, where n is a +ve integer.

   Hence evaluate I_6.

   (b) Evaluate $\int_0^5 \int_0^{x^2} x(x^2+y^2)\, dx\, dy$

Contd.... 2

(7+7+6 Marks)

(c) Evaluate $\int_{-1}^{+1}\int_{0}^{z}\int_{x-z}^{x+z}(x+y+z)\,dx\,dy\,dz$.

5. (a) Define Beta and Gamma functions and show that $\beta(m,n) = \beta(n,m)$.

(b) Show that

i) $\Gamma(n+1) = n\Gamma(n)$

ii) $\Gamma(\frac{1}{2}) = \sqrt{\pi}$

(c) Express the integral in terms of Gamma function $\int_{0}^{\frac{\pi}{2}} \sqrt{\tan\theta}\,d\theta$. (7+7+6 Marks)

6. (a) Solve : $y^2 + x^2\frac{dy}{dx} = xy\frac{dy}{dx}$.

(b) Solve : $\frac{dy}{dx} + \frac{\cos x}{1+\sin x}y = \frac{x}{1+\sin x}$. (7+7+6 Marks)

(c) Solve : $\frac{dy}{dx} = (4x+y+1)^2$.

7. (a) Solve : $(D^2 - 2D + 4)y = e^x \cos x$

(b) Solve : $\frac{d^3y}{dx^3} + 4\frac{dy}{dx} = \sin 2x$. (7+7+6 Marks)

(c) Solve : $(D^3 + D^2 + 4D + 4)y = 0$.

8. (a) Reduce $1 - \cos\alpha + i\sin\alpha$ to the modulus amplitude form.

(b) If $Z\cos\theta = x + \frac{1}{x}$, then prove that $2\cos n\theta = x^n + \frac{1}{x^n}$.

(c) Prove that $\left(\frac{\cos\theta + i\sin\theta}{\sin\theta + i\cos\theta}\right)^4 = \cos 8\theta + \sin 8\theta$. (7+7+6 Marks)

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MATDIP301

Reg. No. 1 V P 0 5 E C 4 0 3

Third Semester B.E Degree Examination, January/February 2006

Common to All Branches

Advanced Mathematics - I

Time: 3 hrs.) (Max.Marks : 100

Note: ***(i) Answer any FIVE full questions.***
(ii) All questions carry equal marks.

1. (a) Obtain the n^{th} derivative of $log(ax+b)$ **(6 Marks)**

   (b) Find the n^{th} derivative of $\frac{x+3}{(x-1)(x+2)}$ **(7 Marks)**

   (c) If $y = sin^{-1}x$, prove that $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - n^2y_n = 0$ **(7 Marks)**

2. (a) With usual notation, prove that

   i) $P = rsin\phi$ ii) $\frac{1}{P^2} = u^2 + \left(\frac{du}{d\theta}\right)^2$ **(5+5 Marks)**

   (b) Find the angle between the radius vector and the tangent of the curve $r = a(1-cos\theta)$ **(5 Marks)**

   (c) Find the pedal equation of the curve $r(1-cos\theta) = 2a$ **(5 Marks)**

3. (a) State Euler's theorem. If $u = \frac{x^3y^3}{x^3+y^3}$, show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 3u$ **(6 Marks)**

   (b) If $u = sin^{-1}(\frac{y}{x})$, verify that $\frac{\partial^2 u}{\partial x \partial y} = \frac{\partial^2 u}{\partial y \partial x}$ **(7 Marks)**

   (c) If $u = x^2, v = y^2$, find $\frac{\partial(u,v)}{\partial(x,y)}$ **(7 Marks)**

4. (a) Obtain the reduction formula $\int_0^{\frac{\pi}{2}} sin^n x dx$ and hence find $\int_0^{\frac{\pi}{2}} sin^4 x dx$ **(5+5 Marks)**

   (b) Evaluate $\int_0^1 \int_{x^2}^{2-x} xy dx dy$ **(5 Marks)**

   (c) Evaluate $\int_0^1 \int_0^x \int_0^{(x+y)^2} x dx dy dz$ **(5 Marks)**

5. (a) Show that $\beta(m,n) = 2\int_0^{\frac{\pi}{2}} sin^{2m-1}\theta . cos^{2n-1}\theta d\theta$ **(6 Marks)**

   (b) Evaluate $\int_0^{\infty} \sqrt{x} e^{-x^3} dx$ **(7 Marks)**

   (c) Prove that $\int_0^{\frac{\pi}{2}} \frac{d\theta}{\sqrt{sin\theta}} \times \int_0^{\pi/2} \sqrt{sin\theta} d\theta = \pi$ **(7 Marks)**

Contd.... 2

6. Solve

(a) $\frac{dy}{dx} = e^{x-y} + x^2e^{-y}$ (5 Marks)

(b) $(x^2 + y^2)dx - 2xydy = 0$ (5 Marks)

(c) $(1 + x^2)\frac{dy}{dx} + xy = sinh^{-1}x$ (5 Marks)

(d) $cos^2x\frac{dy}{dx} + y = tanx$ (5 Marks)

7. Solve

(a) $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = e^{3x}$ (6 Marks)

(b) $\frac{d^2y}{dx^2} + 6y = sin4x$ (7 Marks)

(c) $(D^2 + 5D + 4)y = 3 - 2x$ (7 Marks)

8. (a) Define complex number. Show that the sum and product of a complex number and its conjugate are both real. (5 Marks)

(b) Put the complex number $(\frac{2+i}{3-i})^2$ into polar form. (5 Marks)

(c) Find the modulus and principal argument of $\frac{1+2i}{1-3i}$ (5 Marks)

(d) Simplify $\frac{(cos3\theta+isin3\theta)^4(cos4\theta-isin4\theta)^5}{(cos4\theta+isin4\theta)^3(cos5\theta+isin5\theta)^{-4}}$ (5 Marks)

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MATDIP301

USN | | | | | | | | | | |

Third Semester B.E. Degree Examination, July 2006

Advanced Mathematics – I

Time: 3 hrs.] [Max. Marks:100

Note: *1. Answer any FIVE full questions.*

1 a. Define modulus and amplitude of a complex number x+iy. **(05 Marks)**

b. Find the modulus and amplitude of $\frac{(3-i\sqrt{2})^2}{1+2i}$. **(05 Marks)**

c. Explain the geometrical representation of complex numbers. **(05 Marks)**

d. Express the complex number $\frac{2-i\sqrt{3}}{1+i}$ in the form a+ib. **(05 Marks)**

2 a. Find the nth derivative of the following:
i) sin(ax+b)
ii) $\frac{1}{ax+b}$
iii) Log(ax+b) **(10 Marks)**

b. If $y = a\cos(\log x) + b\sin(\log x)$, prove that $x^2 y_{n+2} + (2n+1)xy_{n+1} + (n^2+1)y_n = 0$. **(10 Marks)**

3 a. With the usual notation, prove that :
i) p = rsinϕ ii) $\frac{1}{p^2} = \frac{1}{r^2} + \frac{1}{r^4}\left(\frac{dr}{d\theta}\right)^2$. **(07 Marks)**

b. Find the pedal equation of the curve r = a(1+cosθ) **(07 Marks)**

c. Using Maclaurin's series, obtain the expression of tanx up to the terms containing x^4. **(06 Marks)**

4 a. If u is a homogenous function of degree n in x and y, then prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = nu$. **(05 Marks)**

b. If $u = x^2+y^2+z^2$, where $x = e^{2t}$, $y = e^{2t}\cos 3t$, $z = e^{2t}\sin 3t$, find $\frac{du}{dt}$ as a total derivative. **(05 Marks)**

c. If u = f (r, s, t), where $r = \frac{x}{y}$, $s = \frac{y}{z}$, $t = \frac{z}{x}$. Prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} = 0$. **(05 Marks)**

d. If x = u(1-v), y = uv. Find $j\left(\frac{x, y}{u, v}\right)$ and $j'\left(\frac{u, v}{x, y}\right)$. Verify that jj' = 1. **(05 Marks)**

Contd... 2

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a. Obtain the reduction formula for $I_n = \int_0^{\pi/2} \sin^n x dx$, n being a positive integer and hence evaluate I_5 and I_6. (07 Marks)

b. Evaluate $\int_0^5 \int_0^{x^2} x(x^2 + y^2) dx dy$. (07 Marks)

c. Evaluate $\int_{-1}^{+1} \int_0^{z} \int_{x-z}^{x+z} (x + y + z) dx dy dz$. (06 Marks)

6 a. Define Beta and Gamma function. (07 Marks)

b. Prove that i) $\Gamma(n+1) = n\Gamma(n)$

ii) $\beta(m,n) = \beta(n,m)$ (07 Marks)

c. Prove that $T\left(\frac{1}{2}\right) = \sqrt{\pi}$. (06 Marks)

7 Solve the following differential equations:

a. $(x + y + 1)^2 \frac{dy}{dx} = 1$. (05 Marks)

b. $ydx - xdy = \sqrt{x^2 + y^2} dx$. (05 Marks)

c. $(x+1)\frac{dy}{dx} - y = e^{3x}(x+1)^2$. (05 Marks)

d. $(x^2 + y^2 - a^2)xdx + (x^2 - y^2 - b^2)ydy = 0$. (05 Marks)

8 Solve the following differential equations:

a. $(D^3 + D^2 + 4D + 4)y = 0$. (05 Marks)

b. $(D^2 + 5D + 6)y = e^x$. (05 Marks)

c. $(D^2 - 2D + 4)y = e^x \cos x$. (05 Marks)

d. $\frac{d^3 y}{dx^3} + 4\frac{dy}{dx} = \sin 2x$. (05 Marks)

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3-02

MATDIP301

Third Semester B.E. Degree Examination, Dec. 07 / Jan. 08

Advanced Mathematics – I

Time: 3 hrs. Max. Marks:100

Note : *Answer any FIVE full questions.*

1 a. Find the n^{th} derivative of – i) $\cos(ax+b)$ ii) $\log(ax+b)$. **(06 Marks)**

b. Find the n^{th} derivative of $\frac{x}{(2x+1)(x+3)}$. **(07 Marks)**

c. If $y=\tan^{-1}x$ prove that : $(1+x^2)y_{n+2}+2(n+1)xy_{n+1}+n(n+1)y_n=0$. **(07 Marks)**

2 a. With usual notation prove that $\tan\phi = r\frac{d\theta}{dr}$. **(06 Marks)**

b. Find the angle between the pairs of curves : $r=6\cos\theta$ $r=2(1+\cos\theta)$. **(07 Marks)**

c. Obtain Maclaurin's series expansion of the function $e^x\sin x$ up to the term containing x^4. **(07 Marks)**

3 a. If $u=\phi(x+ay)+\Psi(x-ay)$, prove that $\frac{\partial^2u}{\partial y^2}=a^2\frac{\partial^2u}{\partial x^2}$. **(07 Marks)**

b. Verify Euler's theorem for the function: $u=x\tan^{-1}\left(\frac{y}{x}\right)$. **(06 Marks)**

c. If $x=r\cos\theta$, $y=r\sin\theta$ find $\frac{\partial(r,\theta)}{\partial(x,y)}$ in terms of r. **(07 Marks)**

4 a. Find the reduction formula for $\int\sin^n x\,dx$. **(06 Marks)**

b. Find the value of $\int_0^1\left(\frac{x^4}{\sqrt{4-x^2}}dx\right)$. **(07 Marks)**

c. Evaluate $\int_0^1\int_{x^2}^{x}(x^2+3y+2)dydx$. **(07 Marks)**

5 a. Prove that $\Gamma\left(\frac{1}{2}\right)=\sqrt{\pi}$. **(06 Marks)**

b. Prove that $\beta(m,n)=2\int_0^{\pi/2}\sin^{2m-1}\theta\cos^{2n-1}\theta\,d\theta$ and hence evaluate $\int_0^{\pi/2}\sqrt{\tan x}\,dx$. **(07 Marks)**

c. Prove that $\int_0^\infty\sqrt{x}e^{-x^2}dx\times\int_0^\infty\frac{e^{-x^2}}{\sqrt{x}}dx=\frac{\pi}{2\sqrt{2}}$. **(07 Marks)**

6 a. Solve $(4x+y+1)^2=\frac{dy}{dx}$ **(06 Marks)**

b. Solve $x^2ydx-(x^3+y^3)dy=0$. **(07 Marks)**

c. Solve $\frac{dy}{dx}=e^{x-y}(e^x-e^y)$. **(07 Marks)**

7 a. Solve $\frac{d^3y}{dx^3}-6\frac{d^2y}{dx^2}+11\frac{dy}{dx}-6y=0$. **(06 Marks)**

b. Solve $\frac{d^{2y}}{dx^2}+4\frac{dy}{dx}+5y=2\cosh x$. **(07 Marks)**

c. Solve $\frac{d^2x}{dx^2}-3\frac{dy}{dx}+2y=\cos 2x$. **(07 Marks)**

8 a. Find the modulus and amplitude of $(1-\cos\alpha+i\sin\alpha)$. **(06 Marks)**

b. Prove that $(1+\cos\theta+i\sin\theta)^n+(1+\cos\theta-i\sin\theta)^n=2^{n+1}\cos^n\frac{\theta}{2}\cos\frac{n\theta}{2}$. **(07 Marks)**

c. Prove that $\sin^7\theta=-\frac{1}{64}(\sin 7\theta-7\sin 5\theta+21\sin 3\theta-35\sin\theta)$. **(07 Marks)**

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MATDIP301

Third Semester B.E. Degree Examination, June/July 08
Advanced Mathematics I

Time: 3 hrs. Max. Marks:100

Note : *Answer any FIVE full questions.*

1 a. Find the modulus and amplitude of $\frac{(3-\sqrt{2}i)^2}{1+2i}$. **(06 Marks)**

b. Express the complex number $\frac{(1-i)(2-i)}{3-i}$ in the form of x + iy. **(07 Marks)**

c. Express the complex number $-1+i\sqrt{3}$ in the polar form. **(07 Marks)**

2 a. If $y = e^{-x}\sinh 3x \cosh 2x$, find y_n. **(06 Marks)**

b. If $y = \tan^{-1} x$, then prove that $(1+x^2)y_{n+2} + 2(n+1)xy_{n+1} + n(n+1)y_n = 0$. **(07 Marks)**

c. Expand $\sin x$ in ascending powers of $\left(x - \frac{\pi}{2}\right)$. **(07 Marks)**

3 a. State Maclaurins theorem and find expansion of e^x. **(06 Marks)**

b. State Taylor's theorem and find the expansion of $\sin x$ in powers of $\left(x - \frac{\pi}{2}\right)$. **(06 Marks)**

c. If $u = e^{x/t^2}$, then prove that $2x\frac{\partial u}{\partial x} + t\frac{\partial u}{\partial t} = 0$. **(08 Marks)**

4 a. If $u = \sin^{-1}\left(\frac{x^2y^2}{x+y}\right)$, prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 3\tan u$. **(06 Marks)**

b. If $u = f(y-z,\ z-x,\ x-y)$, prove that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$. **(07 Marks)**

c. If $x = u(1+v)$, $y = v(1+u)$, show that $\frac{\partial(x,\ y)}{\partial(u,\ v)} = 1+u+v$. **(07 Marks)**

5 a. Derive the reduction formula for $\int \sin^n x\,dx$, where n is + ve integer. **(06 Marks)**

b. Evaluate $\int_0^1 x(1-x^2)^{1/2}dx$. **(07 Marks)**

c. Evaluate $\int_0^1 \int_{x^2}^{2-x^2} xy\,dx\,dy$ **(07 Marks)**

6 a. Evaluate $\int_{-1}^{1}\int_0^z\int_{x-z}^{x+z}(x+y+z)\,dx\,dy\,dz$ **(06 Marks)**

b. Prove that $\beta(m,n) = \frac{\Gamma m \Gamma n}{\Gamma m+n}$, m, n > 0. **(08 Marks)**

c. Evaluate $\int_0^{\frac{\pi}{2}} \sqrt{\cot\theta}\,d\theta$ by expressing in terms of gamma functions. **(06 Marks)**

7 a. Solve $\frac{dy}{dx} = (4x+y+1)^2$. **(06 Marks)**

b. Solve $(x^2-y^2)dx = 2xydy$. **(07 Marks)**

c. Solve $(e^y+1)\cos x\,dx + e^y \sin x\,dy = 0$. **(07 Marks)**

8 a. Solve $x\frac{dy}{dx} + y = x^3y^6$. **(06 Marks)**

b. Solve $(D^3-1)y = 0$. **(07 Marks)**

c. Solve $(D^3 - 6D^2 + 5D)y = (5+x^2)$. **(07 Marks)**

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MATDIP301

Third Semester B.E. Degree Examination, June-July 2009

Advanced Mathematics - I

Time: 3 hrs. Max. Marks:100

Note: ***Answer any FIVE full questions.***

1 a. Find the n^{th} derivative of :

i) $e^{-x} \sin^2 x$; ii) $\log \sqrt{\dfrac{(3x+2)^4 (5-2x)}{(x+2)^3}}$. **(06 Marks)**

b. Find the n^{th} derivative of $\dfrac{x}{(x-1)(2x+3)}$. **(07 Marks)**

c. If $y = \sin^{-1} x$, prove that

i) $(1-x^2)y_2 - xy_1 = 0$

ii) $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - n^2 y_n = 0$. **(07 Marks)**

2 a. If ϕ be the angle between radius vector and tangent, then prove that $\tan\phi = \dfrac{r d\theta}{dr}$. **(06 Marks)**

b. Prove that the curves $r = a(1+\cos\theta)$ and $r = b(1-\cos\theta)$ intersect at right angles. **(07 Marks)**

c. Find the pedal equation of the curve $r^2 = a^2 \sin^2\theta$. **(07 Marks)**

3 a. State and prove Euler's theorem. **(06 Marks)**

b. If $u = F(x-y, y-z, z-x)$, prove that $\dfrac{\partial u}{\partial x} + \dfrac{\partial u}{\partial y} + \dfrac{\partial u}{\partial z} = 0$. **(07 Marks)**

c. If $x = u(1-v)$, $y = uv$, Prove that $\dfrac{\partial(x,y)}{\partial(u,v)} \times \dfrac{\partial(u,v)}{\partial(x,y)} = 1$. **(07 Marks)**

4 a. Obtain the reduction formula for $\int \cos^n x\,dx$. **(06 Marks)**

b. Evaluate $\int_0^a \dfrac{x^7}{\sqrt{a^2 - x^2}}\,dx$. **(07 Marks)**

c. Evaluate $\int_1^2 \int_1^3 xy^2\,dx\,dy$. **(07 Marks)**

5 a. Show that $\Gamma\left(\tfrac{1}{2}\right) = \sqrt{\pi}$. **(06 Marks)**

b. Prove that $\beta(m,n) = \dfrac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$. **(07 Marks)**

c. Express the integral $\int_0^1 \dfrac{dx}{\sqrt{1-x^4}}$ in terms of gamma function. **(07 Marks)**

6 a. Solve : $\dfrac{dy}{dx} = e^{2x-3y} + 4x^2 e^{-3y}$. **(06 Marks)**

b. Solve : $(x^2 - y^2)\,dx = 2xy\,dy$. **(07 Marks)**

c. Solve : $(x^2 - ay)\,dx = (ax - y^2)\,dy$. **(07 Marks)**

7 a. Solve : $\dfrac{d^3y}{dx^3} - 3\dfrac{d^2y}{dx^2} + 3\dfrac{dy}{dx} - y = 0$. **(06 Marks)**

b. Solve : $\dfrac{d^2y}{dx^2} + 4\dfrac{dy}{dx} + 5y = 2\sinh x$. **(07 Marks)**

c. Solve : $\dfrac{d^2y}{dx^2} + 2\dfrac{dy}{dx} + 3y = \sin x$. **(07 Marks)**

8 a. Find the modulus and amplitude of $\dfrac{(3-\sqrt{2}i)^2}{1+2i}$. **(06 Marks)**

b. Prove that $\left(\dfrac{1+\sin\theta + i\cos\theta}{1+\sin\theta - i\cos\theta}\right)^n = \cos\left(\dfrac{n\pi}{2} - n\theta\right) + i\sin\left(\dfrac{n\pi}{2} - n\theta\right)$. **(07 Marks)**

c. Prove that $\cos^6\theta = 1/32\,[\cos 6\theta + 6\cos 4\theta + 15\cos 2\theta + 10]$. **(07 Marks)**

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MATDIP301

Third Semester BE Degree Examination, Dec.09-Jan.10
Advanced Mathematics – I

Time: 3 hrs. Max. Marks:100

Note: ***Answer any FIVE full questions.***

1 a. Find the modulus and amplitude of $\frac{4+2i}{2-3i}$. **(06 Marks)**

b. Express the complex number $2 + 3i + \frac{1}{1-i}$ in the form of a + ib. **(07 Marks)**

c. Express the complex number $\sqrt{3}+i$ in the polar form. **(07 Marks)**

2 a. Find the n^{th} derivative of $e^{-x}\sin^2 x$. **(06 Marks)**

b. Find the n^{th} derivative of $\frac{x}{(x-1)(2x+3)}+e^{2x}$. **(07 Marks)**

c. If $y = \sin^{-1}x$ then prove that $(1 + x^2)\, y_{n+2} - (2n + 1)\, xy_{n+1} - n^2 y_n = 0$. **(07 Marks)**

3 a. Using Maclaurin's series expand tanx upto the term containing x^3. **(06 Marks)**

b. Find the angle between the radius vector and tangent to the curve $r = \sin\theta + \cos\theta$. **(07 Marks)**

c. With usual notations prove that

i) $P = r\sin\phi$

ii) $\frac{1}{p^2}=\frac{1}{p^2}+\frac{1}{p^4}\left(\frac{dr}{d\theta}\right)^2$. **(07 Marks)**

4 a. If $u = f(r, s, t)$ where $r=\frac{x}{y}$, $s=\frac{y}{z}$, $t=\frac{z}{x}$ then prove that $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}+z\frac{\partial u}{\partial z}=0$. **(06 Marks)**

b. If $u = f(x + ay) + g(x - ay)$ then show that $\frac{\partial^2 u}{\partial y^2}=a^2\frac{\partial^2 u}{\partial x^2}$. **(07 Marks)**

c. If $u = x^2 - 2y$, $v = x + y + z$, $w = x - 2y + 3z$ then find $\frac{\partial(u,v,w)}{\partial(x,y,z)}$. **(07 Marks)**

5 a. Obtain the reduction formula for $\int \cos^n x\,dx$ where n is a positive integer. **(06 Marks)**

b. Evaluate $\int_0^1 x^6\sqrt{1-x^2}\,dx$. **(07 Marks)**

c. Evaluate $\int_0^1\int_x^{\sqrt{x}}(x^2+y^2)\,dydx$.. **(07 Marks)**

6 a. Evaluate $\int_0^1\int_0^2\int_1^2 x^2yz \ \ dzdydx$. (06 Marks)

b. Prove that $\sqrt{\frac{1}{2}} = \sqrt{\pi}$. (07 Marks)

c. Show that $\int_0^{\pi/2}\sqrt{\text{Sin}\theta}\ \ d\theta \ \ x \int_0^{\pi/2}\frac{d\theta}{\sqrt{\text{Sin}\theta}} = \pi$. (07 Marks)

7 a. Solve $(e^y + 1)\ \text{Cos}xdx + e^y\ \text{Sin}x\ dy = 0$. (06 Marks)

b. Solve $ydx - xdy = \sqrt{x^2 + y^2}\ dx$. (07 Marks)

c. Solve $x\frac{dy}{dx} + y = x^3y^6$. (07 Marks)

8 a. Solve $4\frac{d^3y}{dx^3} + 4\frac{d^2y}{dx^2} + \frac{dy}{dx} = 0$. (06 Marks)

b. Solve $\frac{d^2x}{dt^2} + 2\frac{dx}{dt} + 3x = \text{Sint} + e^{-t}$. (07 Marks)

c. Solve $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = xe^x \sin x$. (07 Marks)

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MATDIP301

Third Semester B.E. Degree Examination, May/June 2010

Advanced Mathematics – I

Time: 3 hrs. Max. Marks:100

Note: ***Answer any FIVE full questions.***

1 a. Express the complex number $\dfrac{(1+i)(1+3i)}{1+5i}$ in the form x + iy. **(06 Marks)**

b. Prove that $(1+i)^n + (1-i)^n = 2^{\frac{n}{2}+1}\cos\left(\dfrac{n\pi}{4}\right)$. **(07 Marks)**

c. Expand $\cos^8\theta$ in a series of cosines multiples of θ. **(07 Marks)**

2 a. Find the n^{th} derivative of $e^{ax}\sin(bx + c)$. **(06 Marks)**

b. If y = a cos (log x) + b sin (log x), prove that $x^2 y_{n+2} + (2n+1)xy_{n+1} + (n^2+1)y_n = 0$. **(07 Marks)**

c. Find the n^{th} derivative of $\dfrac{x}{(x-1)(2x+3)}$. **(07 Marks)**

3 a. State Taylor's theorem and expand the polynomial $2x^3 + 7x^2 + x - 6$ in powers of (x -1). **(06 Marks)**

b. Expand tan x in ascending powers of x using MacLaurin's theorem upto the term containing x^4. **(07 Marks)**

c. If $Z = \dfrac{x^2+y^2}{x+y}$ prove that $\left(\dfrac{\partial z}{\partial x} - \dfrac{\partial z}{\partial y}\right)^2 = 4\left(1 - \dfrac{\partial z}{\partial x} - \dfrac{\partial z}{\partial y}\right)$. **(07 Marks)**

4 a. If $u = \tan^{-1}\left(\dfrac{x^3+y^3}{x-y}\right)$, prove that $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} = \sin 2u$. **(06 Marks)**

b. If u = f(x, y) where x = r cos θ and y = r sin θ, prove that $\left(\dfrac{\partial u}{\partial x}\right)^2 + \left(\dfrac{\partial u}{\partial y}\right)^2 = \left(\dfrac{\partial u}{\partial r}\right)^2 + \dfrac{1}{r^2}\left(\dfrac{\partial u}{\partial \theta}\right)^2$. **(07 Marks)**

c. If $u = x^2 - 2y$, $v = x + y + z$ and $w = x - 2y + 3z$, find the value of $J\left(\dfrac{u, v, w}{x, y, z}\right)$. **(07 Marks)**

5 a. Obtain the reduction formula for $\int \sin^m x \cos^n x\, dx$. **(06 Marks)**

b. Evaluate $\displaystyle\int_0^a \frac{x^7}{\sqrt{a^2 - x^2}}\, dx$. **(07 Marks)**

c. Evaluate $\displaystyle\int_0^1\int_0^x e^{\left(\frac{y}{x}\right)} dy\, dx$. **(07 Marks)**

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

6 a. Evaluate $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} xyz\,dz\,dy\,dx$. **(06 Marks)**

b. Prove that $\beta(m,n) = \dfrac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$. **(07 Marks)**

c. Show that $\int_0^{\pi/2} \sqrt{\sin\theta}\,d\theta \times \int_0^{\pi/2} \dfrac{d\theta}{\sqrt{\sin\theta}} = \pi$. **(07 Marks)**

7 a. Solve $3e^x \tan y\,dx + (1 - e^x)\sec^2 y\,dy = 0$. **(06 Marks)**

b. Solve $x^2 y\,dx = (x^3 + y^3)\,dy$. **(07 Marks)**

c. Solve $x\dfrac{dy}{dx} + y = x^3 y^6$. **(07 Marks)**

8 a. Solve $\dfrac{d^2y}{dx^2} + \dfrac{dy}{dx} = x^2 + 2x + 4$. **(06 Marks)**

b. Solve $\dfrac{d^2y}{dx^2} + 3\dfrac{dy}{dx} + 2y = 4\cos^2 x$. **(07 Marks)**

c. Solve $\dfrac{d^3y}{dx^3} + 2\dfrac{d^2y}{dx^2} + \dfrac{dy}{dx} = e^{-x} + \sin 2x$. **(07 Marks)**

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